

Formation of Differential Equations

1 Mark Questions

1. Write the degree of the differential equation

$$\left(\frac{dy}{dx}\right)^4 + 3y \frac{d^2y}{dx^2} = 0.$$

Delhi 2013C



The degree of the differential equation is the degree of the highest order derivative, when differential coefficients are made free from radicals and fractions sign.

Given differential equation is

$$\left(\frac{dy}{dx}\right)^4 + 3y \frac{d^2y}{dx^2} = 0$$

Here, highest order derivative is d^2y/dx^2 , whose degree is one. So, degree of differential equation is 1. (1)

2. Write the degree of the differential equation

$$x^3 \left(\frac{d^2y}{dx^2}\right)^2 + x \left(\frac{dy}{dx}\right)^4 = 0.$$

Delhi 2013

Given differential equation is

$$x^3 \left(\frac{d^2y}{dx^2}\right)^2 + x \left(\frac{dy}{dx}\right)^4 = 0$$

Here, all differential coefficients are free from radical sign.

∴ Degree = 2 (1)

3. Write the degree of the differential equation

$$\left(\frac{dy}{dx}\right)^4 + 3x \frac{d^2y}{dx^2} = 0.$$

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Given differential equation is

$$\left(\frac{dy}{dx}\right)^4 + 3x \left(\frac{d^2y}{dx^2}\right) = 0$$



Here, all differential coefficients are free from radical sign.

$$\therefore \text{Degree} = 1 \quad (1)$$

4. Write the differential equation representing the family of curves $y = mx$, where m is an arbitrary constant. All India 2013

Given, family of curves is $y = mx$... (i)

where, m is an arbitrary constant.

Now, differentiating Eq. (i) w.r.t. x , we get

$$\frac{dy}{dx} = m$$

On putting $m = \frac{dy}{dx}$ in Eq. (i), we get

$$y = x \frac{dy}{dx}$$

which is the required differential equation. (1)

4 Marks Questions

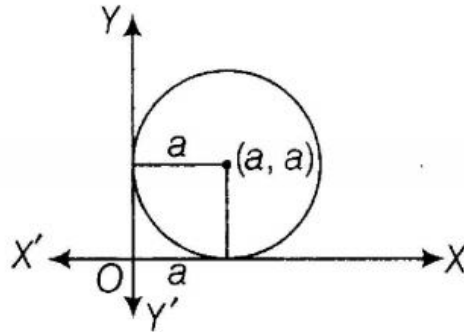
5. Find the differential equation of the family of circles in the first quadrant which touch the coordinate axes. All India 2010C





The equation of family of circles in first quadrant, which touch the coordinate axes, is $(x-a)^2 + (y-a)^2 = a^2$, where a is radius of circle. Differentiate it one time and eliminate the arbitrary constant a .

Let a be the radius of family of circles in the first quadrant, which touch the coordinate axes.



Then, coordinates of centre of circle = (a, a) .

(1)

We know that, equation of circle which has centre (h, k) and radius r is given by

$$(x - h)^2 + (y - k)^2 = r^2$$

Here, $(h, k) = (a, a)$ and $r = a$

∴ Equation of family of such circles is

$$(x - a)^2 + (y - a)^2 = a^2 \quad \dots(i) \quad (1)$$

On differentiating both sides w.r.t. x , we get

$$2(x - a) + 2(y - a) \frac{dy}{dx} = 0$$

$$\Rightarrow x - a + (y - a) \cdot y' = 0 \quad \left[\because \frac{dy}{dx} = y' \right]$$

$$\Rightarrow x + yy' = a + ay'$$

$$\Rightarrow a = \frac{x + yy'}{1 + y'} \quad (1)$$



On putting above value of a in Eq. (i), we get

$$\begin{aligned} \left[x - \frac{x + yy'}{y' + 1} \right]^2 + \left[y - \frac{x + yy'}{y' + 1} \right]^2 &= \left(\frac{x + yy'}{y' + 1} \right)^2 \\ \Rightarrow \left[\frac{xy' + x - x - yy'}{y' + 1} \right]^2 + \left[\frac{yy' + y - x - yy'}{y' + 1} \right]^2 &= \left(\frac{x + yy'}{y' + 1} \right)^2 \end{aligned}$$

On multiplying both sides by $(y' + 1)^2$, we get

$$\begin{aligned} (xy' - yy')^2 + (y - x)^2 &= (x + yy')^2 \\ \Rightarrow (x - y)^2 (y')^2 + (x - y)^2 &= (x + yy')^2 \\ &[\because (x - y)^2 = (y - x)^2] \\ \Rightarrow (x - y)^2 [(y')^2 + 1] &= (x + yy')^2 \end{aligned}$$

which is the required differential equation. **(1)**

- 6.** Find the differential equation of family of circles touching Y-axis at the origin.

HOTS; Delhi 2010; All India 2009, 2008C



The equation of family of circles touching Y-axis at origin is given by $(x - a)^2 + y^2 = a^2$, where a is radius of circle.

Differentiate this equation once because one arbitrary constant is present in the equation and eliminate a .

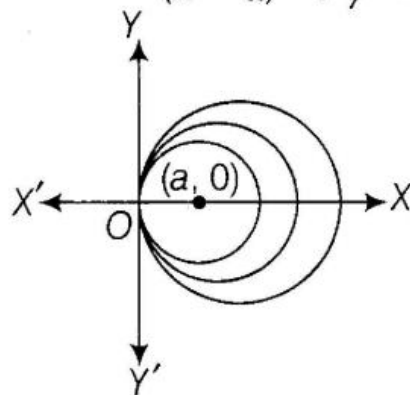
Given, family of circles touch Y-axis at the origin.

Let radius of family of circles be a .

∴ Centre of circle = $(a, 0)$ (1)

Now, equation of family of circles with centre $(a, 0)$ and radius a is

$$(x - a)^2 + y^2 = a^2 \quad (1)$$



[putting $(h, k) = (a, 0)$ and $r = a$

$$\text{in } (x - h)^2 + (y - k)^2 = r^2]$$

$$\Rightarrow x^2 + a^2 - 2ax + y^2 = a^2$$

$$\Rightarrow x^2 - 2ax + y^2 = 0 \quad \dots(i)$$

On differentiating both sides w.r.t. x , we get

$$2x - 2a + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow x^2 - 2ax + y^2 = 0 \quad \dots(i)$$

On differentiating both sides w.r.t. x , we get

$$2x - 2a + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow a = x + y \frac{dy}{dx} \quad (1)$$

On putting above value of a in Eq. (i), we get

$$x^2 + y^2 - 2 \left(x + y \frac{dy}{dx} \right) x = 0$$

$$\Rightarrow 2xy \frac{dy}{dx} + x^2 - y^2 = 0$$

$$\Rightarrow 2xyy' + x^2 - y^2 = 0$$

$$\text{or } 2xy \frac{dy}{dx} + x^2 - y^2 = 0$$

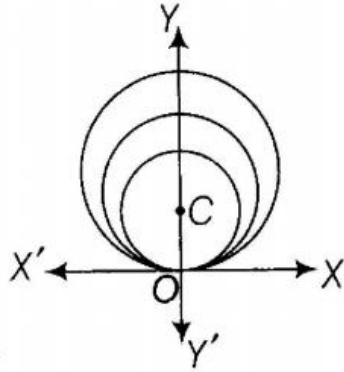
which is the required differential equation. (1)

7. Find the differential equation of family of circles touching X-axis at the origin.

HOTS; Delhi 2010C; All India 2009C



Let a be the radius of family of circles which touch X -axis at origin.



\therefore Centre of circle = $(0, a)$

Now, equation of family of such circles is

$$x^2 + (y - a)^2 = a^2 \quad (1)$$

[putting $(h, k) = (0, a)$ and $r = a$
in $(x - h)^2 + (y - k)^2 = r^2$]

$$\Rightarrow x^2 + y^2 - 2ay = 0 \quad \dots(i)$$

On differentiating both sides w.r.t. x , we get

$$2x + 2y \frac{dy}{dx} - 2a \frac{dy}{dx} = 0$$

$$\Rightarrow x + (y - a) \frac{dy}{dx} = 0 \quad (1)$$

$$\Rightarrow x + yy' - ay' = 0, \left[\text{where, } y' = \frac{dy}{dx} \right]$$

$$\Rightarrow x + yy' = ay'$$

$$\Rightarrow a = \frac{x + yy'}{y'} \quad (1)$$

On putting above value of a in Eq. (i), we get

$$x^2 + y^2 = 2y \left(\frac{x + yy'}{y'} \right)$$

$$\Rightarrow (x^2 + y^2) \cdot y' = 2xy + 2y^2 \cdot y'$$

$$\Rightarrow x^2y' + y^2y' - 2xy - 2y^2y' = 0$$

$$\Rightarrow x^2y' - 2xy - y^2y' = 0$$

$$\Rightarrow y'(x^2 - y^2) = 2xy$$

$$\Rightarrow y' = \frac{2xy}{x^2 - y^2} \quad \text{or} \quad \frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

which is the required differential equation. (1)

8. Form the differential equation representing family of ellipses having foci on X-axis and centre at the origin. **HOTS; Delhi 2009C**

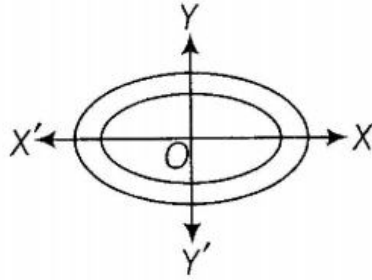




The equation of family of ellipses having foci on X-axis and centre at origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$.

Differentiate this equation two times and eliminate two arbitrary constants a and b to get the required result.

We know that, the equation of family of ellipse having foci on X-axis and centre at origin is given by



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \dots(i)$$

where, $a > b$ **(1)**

On differentiating both sides of Eq. (i) w.r.t. x , we get

$$\frac{2x}{a^2} + \frac{2yy'}{b^2} = 0 \quad \left[\text{put } \frac{dy}{dx} = y' \right]$$

$$\Rightarrow \frac{x}{a^2} = \frac{-yy'}{b^2}$$

$$\Rightarrow \frac{yy'}{x} = \frac{-b^2}{a^2} \quad \dots(ii) \text{ (1)}$$

Again, differentiating both sides of Eq. (ii) w.r.t. x , we get

$$\left[\frac{x \cdot \frac{d}{dx}(yy') - yy' \cdot \frac{d}{dx}(x)}{x^2} \right] = 0$$

$$\left[\begin{array}{l} \text{using quotient rule of differentiation} \\ \text{in LHS and } \frac{d}{dx} \left(\frac{-b^2}{a^2} \right) = 0 \end{array} \right]$$

$$\Rightarrow x \left[y \cdot \frac{d}{dx}(y') + y' \cdot \frac{d}{dx}(y) \right] - yy' \cdot 1 = 0 \quad \text{(1)}$$

$$\Rightarrow \quad xyy'' + x(y')^2 - yy' = 0$$

$$\text{or} \quad xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$$

which is the required differential equation. **(1)**

- 9.** Form the differential equation representing family of curves given by $(x - a)^2 + 2y^2 = a^2$ where, a is an arbitrary constant. **All India 2009**

Given equation of family of curves is

$$(x - a)^2 + 2y^2 = a^2 \quad \dots(i)$$

On differentiating both sides w.r.t. x in Eq. (i), we get

$$2(x - a) + 4yy' = 0 \quad \left[\because \frac{d}{dx}(y^2) = 2yy' \right] \quad (1)$$

$$\Rightarrow \quad x - a + 2yy' = 0$$

$$\Rightarrow \quad a = x + 2yy' \quad (1)$$

On putting above value of a in Eq. (i), we get

$$(x - x - 2yy')^2 + 2y^2 = (x + 2yy')^2$$

$$\Rightarrow \quad 4y^2(y')^2 + 2y^2 = x^2 + 4y^2(y')^2 + 4xyy'$$

$$\Rightarrow \quad 2y^2 = x^2 + 4xyy' \quad (1)$$

Hence, the required differential equation is

$$x^2 + 4xyy' = 2y^2$$

$$\text{or} \quad x^2 + 4xy \frac{dy}{dx} = 2y^2 \quad (1)$$